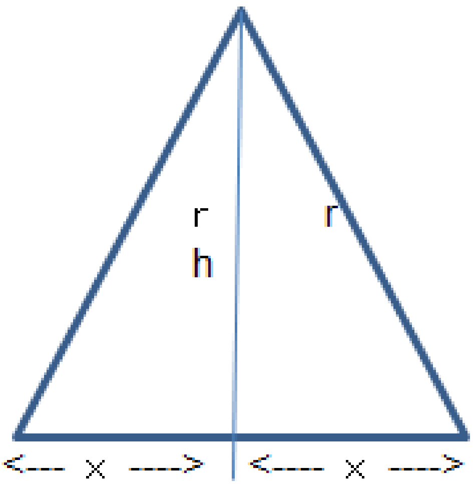
A quick note on business calculus

"The area of a figure formed by a string of length L "

What shape has the largest area of a shape made from a string of length L? What type are triangles and rectangles? How does it compare to the circle ?

1. For triangles: equilateral triangles have the largest area

$$S = \frac{L^2}{12\sqrt{3}}$$



Assuming the length of the rope

r + r + 2x =L   r=L/2-x

$$h = \sqrt{r^2 - x^2} = \sqrt{\left(\frac{L}{2} - x\right)^2 - x^2} = \sqrt{\frac{L^2}{4} + x^2 - Lx - x^2} = \sqrt{\frac{L^2}{4} - Lx}$$

$$S = \frac{1}{2} \cdot h \cdot x \cdot 2 = hx = x\sqrt{\frac{L^2}{4} - Lx}$$ To differentiate this

$$S' = (x)'\sqrt{\frac{L^2}{4} - Lx} + x\left(\sqrt{\frac{L^2}{4} - Lx}\right)'$$

$$= \sqrt{\frac{L^2}{4} - Lx} + x\left(-\frac{L}{2} \cdot \frac{1}{\sqrt{\frac{L^2}{4} - Lx}}\right) = \frac{L\left(\frac{L}{4} - \frac{3}{2}x\right)}{\sqrt{\frac{L^2}{4} - Lx}}$$

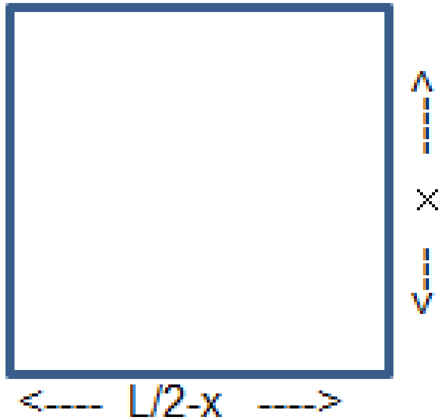
$$\sqrt{\frac{L^2}{4} - Lx} > 0 \quad \therefore \frac{L}{4} - \frac{3}{2}x$$ S is maximum

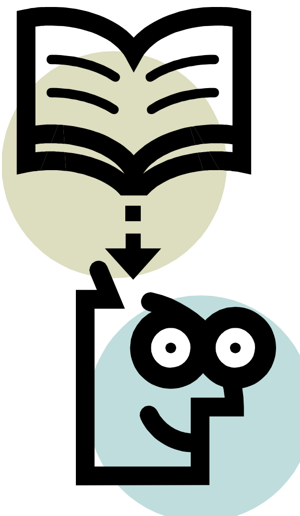
$$\frac{L}{4} = \frac{3}{2}x \qquad x = \frac{1}{6}L$$

$$\therefore r = \frac{L}{2} - x = \frac{L}{2} - \frac{L}{6} = \frac{1}{3}L$$ ← Equilateral triangle

$$S = \frac{1}{6}L\sqrt{\frac{L^2}{4} - \frac{1}{6}L^2} = \frac{1}{6}L^2\sqrt{\frac{1}{12}} = \frac{L^2}{12\sqrt{3}}$$

2. For rectangles: squares have the largest area $S = \frac{L^2}{16}$





Assuming the length of the rope is L,

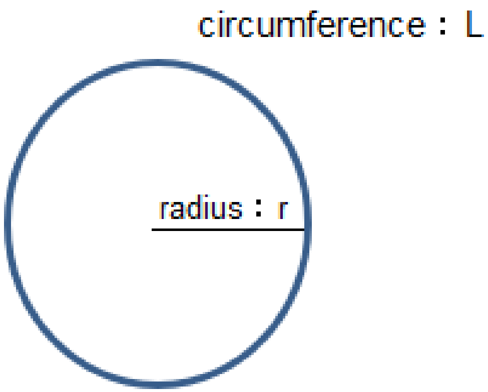
$S = x(\frac{L}{2} - x) = \frac{1}{2}Lx - x^2$

If we differentiate this $S' = \frac{L}{2} - 2x$

$x = \frac{L}{4}$ therefore square is maximum area

$S = \frac{L}{4}(\frac{L}{2} - \frac{L}{4}) = \frac{L^2}{16}$

3. For circles: $S=\frac{L^2}{4\pi}$



$S=\pi r^2$ $L=2\pi r$ $r=\frac{L}{2\pi}$

$S=\pi(L/2\pi)^2=L^2/4\pi$

4. evaluation

shape	maximum area type	area	area when L=1
triangle	equilateral triangle	$S=\frac{L^2}{12\sqrt{3}}$	0.0481
rectangle	square	$S=\frac{L^2}{16}$	0.0625
circle	N/A	$S=\frac{L^2}{4\pi}$	0.0796

①The area increases in the order of
circle > square > equilateral triangle
as the angles that can be taken become larger.

②Nature is beautiful and seeks harmony,
equilateral triangle for triangle, square for rectangle.